

Normal Distributions

- It is a continuous theoretical distribution in Statistics.
- It was first discovered by English mathematician De-moivre (1667-1754) in 1733.
- Normal distribution is also known as Gauss's distribution (Gaussian Law of Errors) after Karl Friedrich Gauss (1777-1855).

Equation of Normal Probability Curve:

If x is a continuous random variable following normal probability distribution with mean μ and standard deviation σ , then its probability density function is

$$P(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty.$$

$$\therefore P(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where,

$$\pi_1 = \frac{22}{7}$$

$$\sqrt{2\pi} = 2.5066$$

$$e = 2.71828$$

The mean μ and standard deviation σ are called the parameters of the normal distribution.

The Standard normal variate Z

$$Z = \frac{X - E(X)}{\sigma_X} = \frac{X - \mu}{\sigma}$$

$$E(Z) = E\left(\frac{X - \mu}{\sigma}\right) = \frac{1}{\sigma} E(X - \mu)$$

$$= \frac{1}{\sigma} [E(X) - E(\mu)] = \frac{1}{\sigma} (\mu - \mu) = 0 \quad [E(cx) = cE(x)]$$

$$\text{Var}(Z) = \text{Var}\left(\frac{X - \mu}{\sigma}\right)$$

$$= \frac{1}{\sigma^2} \text{Var}(X - \mu)$$

$$= \frac{1}{\sigma^2} \text{Var}(X) \quad [\text{Var}(cx) = c^2 \text{Var}(x)]$$

$$\text{Var}(Z) = \frac{1}{\sigma^2}, \quad \sigma^2 = 1$$

\therefore Hence, Standard normal variate Z has mean 0 and Standard deviation 1.

The Probability density function of Standard Normal variate Z is given by

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty$$

This gives the height (ordinate) of standard normal curve at the point z .

Reference:

Fundamentals of statistics by S.C. Gupta.